

CS 383

HW 4

Due in class Wednesday, March 1

This one should be typed.

1. Remember quotient languages from HW 3: If L is a regular language over Σ and $a \in \Sigma$ then L/a is the set of strings w such that wa is in L . Either prove or disprove the following identities:
 - a. $(L/a)a = L$
 - b. $(La)/a = L$
2. Suppose L is a regular language. Show that $\text{min}(L)$ is also regular, where $\text{min}(L) = \{w \mid w \text{ is in } L \text{ but no proper prefix of } w \text{ is in } L\}$
3. Suppose L is regular. Show that $\text{prefix}(L)$ is also regular, where $\text{prefix}(L) = \{w \mid wx \text{ is in } L \text{ for some } x \text{ (including } x=\epsilon)\}$. $\text{prefix}(L)$ is the set of all prefixes of all strings in L . These don't need to be proper prefixes, so L is a subset of $\text{prefix}(L)$
4. For any language L let $\text{powers}(L) = \{x^n \mid n \geq 0 \text{ and } x \in L\}$. Find an example where L is regular but $\text{powers}(L)$ is not regular.
5. Design a context-free grammar for $\{0^n 1^n \mid n \geq 1\}$
6. Design a context-free grammar for $\{a^i b^j c^k \mid i \neq j\}$
7. Here is a context-free grammar:

$$S \Rightarrow aS \mid Sb \mid a \mid b$$

Prove by induction on the string length that no string in the language represented by this grammar has ba as a substring.